The Imperfect Information Flow of Agents Communication in Arrow Logic

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Abstract

This paper formalizes the communication of agents with belief modal operators. First we investigate the logic of knowledge, belief, and perception especially for conjunction / disjunction decomposability. Thereafter, we propose the information flow model, where knowledge is regarded as a site, perception as a channel, and belief as parts of sentences. We check the satisfiability of logically connected modal sentences, using the test principles by Barwise on this information flow model. Then we expand our model to the agents-network model by arrow logic. Lastly, we summarize the communication of agents, and discuss the applicability of our model.

1. Introduction

Over the last few years, a lot of studies have been made on the logical formalization of rational agents ((Moore, 1985; Wooldridge and Jennings, 1995; Wooldridge, Muller, and Tambe, 1996; Muller, Wooldridge, and Jennings, 1997; van Linder, van der Hoek, and Meyer, 1997), and so on). According to (Shoham and Cousins, 1994), there are two main approaches in this field: one is an informational aspect such as epistemic modal logic like logic of knowledge and belief (Halpern and Moses, 1992), and the other is a motivational aspect such as commitments and obligations ((Cohen and Levesque, 1986; Cohen and Levesque, 1990), and so on). These formalizations mainly concern how agents act and/or change their internal states after they receive information, while interactions between agents seem to have been rather neglected. In this paper, we propose a formalization of communication channels between agents, as well as that of communicative agents.

Thus far, the behavior of agents, embedded in a situation, with such modal operators as B (belief), D(desire), and I(intention), has been investigated by many reserchers ((Rao and Georgeff, 1995; Shoham and del Val, 1994), and so on). Devlin (Devlin, 1991) has rather independently investigated the nature of B (belief), D (desire), S (see), and ST (see that) in natural language. The objective of this paper is to propose a model of the information flow between agents. Thus, agents who have knowledge and belief, communicate with other agents, are modeled from the information theoretic point of view (Barwise, Gabbay, and Hartonas, 1996). Our approach is similar to (van Linder, van der Hoek, and Meyer, 1997) and (Moss and Seligman, 1994), however, we mainly pay attention to imperfect information flow. In (van Linder, van der Hoek, and Meyer, 1997), the organization of believes in agents' internal states is examined. The channel algebra which is almost the same as the arrow logic is proposed in (Moss and Seligman, 1994). In this paper, we investigate the communication with modal operator like belief between agents. To realize neat and clear formalization, we adopt arrow logic and propositional dynamic logic.

In the following section, we investigate the nature of modal operators that appear in natural languages, based on (Devlin, 1991). Then, we formalize the role of modal operators in term of information theory (Barwise, Gabbay, and Hartonas, 1996). Thereafter, we apply the test principles of Barwise (Barwise, 1993) to our communication model. Then we define channels as arrows and iteration of channels by arrow logic and dynamic logic, and discuss in the final section.

2. Logic of belief and perception

In this section, we formalize the logic of belief, desire, and perception by Devlin(Devlin, 1991). We define the following operators.

$B_a\Phi$: agent a believes Φ.

$D_a\Phi$: agent a desires Φ.

$S_a\Phi$: agent a sees Φ.

$ST_a\Phi$: agent a sees that Φ.

We cannot claim the veridicality of a sentence if it is accompanied by B operator. Also, we cannot decompose such a B sentence into logical 'or' subsentences. Below are the examples by Devlin (Devlin, 1991). (We omit the subscript of agent name in operators in the following examples.)

> (1) "That Naomi believes Melissa ate the cookie does not entail that Melissa actually did eat the cookie."
> (2) "Naomi can believe we will spend the night in Monterey or we will spend the night in Carmel, without believing that we will spend the night in Monterey and without believing we will spend the night in Carmel."

Both of the examples are formalized in the following way:

$(1)'$ $B\Phi \not\supset \Phi$
$(2)'$ $B(\Phi \vee \Psi) \not\supset (B\ \Phi \vee B\ \Psi)$

where '$\supset$' is the logical implication.

In the similar way, we have investigated natural language sentences including B, D, S, and ST operators, and their logical features are summarized as in the following table.

	Veridicality	Conjunction	Disjunction
Belief	$B\Phi \not\supset \Phi$	$B(\Phi \wedge \Psi) \supset (B\Phi \wedge B\Psi)$	$B(\Phi \vee \Psi) \not\supset (B\ \Phi \vee B\ \Psi)$
		$B(\Phi \wedge \Psi) \not\subset (B\Phi \wedge B\Psi)$	$B(\Phi \vee \Psi) \not\subset (B\ \Phi \vee B\ \Psi)$
Desire	$D\Phi \not\supset \Phi$	$D(\Phi \wedge \Psi) \supset (D\Phi \wedge D\Psi)$	$D(\Phi \vee \Psi) \supset (D\ \Phi \vee D\ \Psi)$
		$D(\Phi \wedge \Psi) \not\subset (D\Phi \wedge D\Psi)$	$D(\Phi \vee \Psi) \not\subset (D\ \Phi \vee D\ \Psi)$
See	$S\Phi \supset \Phi$	$S(\Phi \wedge \Psi) \supset (S\ \Phi \wedge S\ \Psi)$	$S(\Phi \vee \Psi) \supset (S\ \Phi \vee S\ \Psi)$
		$S(\Phi \wedge \Psi) \subset (S\Phi \wedge S\Psi)$	$S(\Phi \vee \Psi) \subset (S\ \Phi \vee S\ \Psi)$
See that	$ST\Phi \supset \Phi$	$ST(\Phi \wedge \Psi) \supset (ST\Phi \wedge ST\Psi)$	$ST(\Phi \vee \Psi) \not\supset (ST\ \Phi \vee ST\ \Psi)$
		$ST(\Phi \wedge \Psi) \not\subset (ST\Phi \wedge ST\Psi)$	$ST(\Phi \vee \Psi) \not\subset (ST\ \Phi \vee ST\ \Psi)$

Thus far, we investigated the feature of 'see' and 'see that' as two different perception methods, and found their veridicality: $S\Phi \supset \Phi$ and $ST\Phi \supset \Phi$. However, the difference seems not to be intrinsic in agent communication. From now on in the latter part of this paper, we generalize S and ST into a more general perception P, requiring P owns the same feature: $P\Phi \supset \Phi$, and no longer mention S and ST. Also, we will disregard the operator D, because D behaves almost same as B, and D's own logical importance seems low. Our position is that B can represent the modality of imperfect information flow in agent communication.

3. The modal operators in the flow of information

The flow of information has been formalized as a constraint in two situation types:

$$\begin{array}{ccc} S_0 & \Rightarrow & S_1 \\ f\downarrow & & f'\downarrow \\ s_0 & & s_1 \end{array}$$

When a situation type S_0 is anchored to a concrete situation s_0 by f, then this constraint is activated as an information flow, viz., S_1 is also anchored to a concrete situation s_1 and information flows through a channel from site s_0 to site s_1 (Barwise, 1993). Our objective here is to regard this information flow as communication between agents. As information through a channel can be considered as a natural language sentence, the information can be a sentence with modal operators formalized in the previous section. These modal operators may concern the flow of information.

When we regard that both sites are the agents, how should we locate the modal operators in the diagram? We can consider the following three alternatives, where O is some operator.

1. An operator O is a **site**. Namely, $O_a\Phi$ is implemented as $O_a \models_S \Phi$ in a site.

2. An operator O is a **channel**. Namely, $O_a\Phi$ is implemented as $\Phi \stackrel{O}{\Rightarrow} \Phi$.

3. An operator is just a **part of a sentence**.

Where $s \models_S \sigma$ means the infon σ is true in the situation s or s supports σ, the infon σ is defined:

Objects $a_1, a_2, \cdots, a_n$ have the relation P or not:

$\ll P, a_1, a_2, \cdots, a_n, 1 \gg$ or $\ll P, a_1, a_2, \cdots, a_n, 0 \gg$

and a situation s is a set of infons $\{\sigma \mid s \models_S \sigma\}$.

3.1 K as Site

We can use an agent name directly as a site name, as another well-known operator K_a by Hintikka (Hintikka, 1962) can clarify the character of sites. Thus, if we are to model communicative agents and the information flow between them, the knowledge of each agent should be the information site. A natural language sentence $K_a\Phi$ becomes a proposition: $\mathcal{K}_a \models_S \Phi$. (We use calligraphic characters for site names, and distinguish them from operators.)

Other kinds of sites can be defined inductively from given $\mathcal{K}$'s.

$$\mathcal{B}_a = \{\Phi \mid \mathcal{K}_a \models_S B\Phi\}.$$

3.2 P as Channel

When an agent utters Φ and another agent perceives $P\Phi$, he/she comes to know Φ as well as $P\Phi$ by its veridicality. Therefore, we can define the perception operator P as a channel. An agent's dispatch of information is to put the information on the other agent's perception channel. We can locate K's and P as:

$$\mathcal{K}_a \models_S \Phi \stackrel{P}{\Rightarrow} \mathcal{K}_b \models_S \Phi,$$

where the agent a knows Φ and says Φ, then the agent b perceives Φ.

3.3 B as part of Sentence

We define other remaining operator, the belief operator B as parts of sentences. Thus, $\mathcal{K}_a \models_S B_a\Phi$ means that "the agent a believes Φ in site $\mathcal{K}_a$."

The interaction between different operator locations is summarized as follows:

Site and Sentence $\mathcal{B}_a \models_S \Phi$ *iff* $\Phi \in \mathcal{B}_a$, so $\mathcal{B}_a \models_S \Phi$ *iff* $\mathcal{K}_a \models_S B_a\Phi$

Site and Channel $\mathcal{B}_a \models_S \Phi$ (*iff* $\mathcal{K}_a \models_S B_a\Phi) \stackrel{P}{\Rightarrow} \mathcal{K}_b \models_S B_a\Phi$

Sentence and Channel $\mathcal{K}_a \models_S B_a\Phi \stackrel{P}{\Rightarrow} \mathcal{K}_b \models_S B_a\Phi$

4. Agent communication through channels

In this section, we discuss several features of agent communication as information flow, to verify the applicability of our model.

4.1 Test principles

As we claimed that agent communication can be modeled in information flow of Barwise(Barwise, 1993), we need to inspect the satisfiability of our model in his test principles. First we look back his definition of information network (Barwise, Gabbay, and Hartonas, 1996).

Information network An information network is a structure of the form $\mathcal{N} =< S, C, \Rightarrow, \circ >$ where S is a set of objects called *sites*, C is a set of objects called channels, $\Rightarrow$ is a relation on $S \times C \times S$, and $\circ$ is an associative binary operation on C. The signaling relation $s \overset{c}{\Rightarrow} t$ is read c is a channel from *source* s to *target* t. A *connection* for the channel c is a pair $< s, t >$ such that $s \overset{c}{\Rightarrow} t$. The channel $a \circ b$ is called the composition of a and b. The signaling relation and composition operation are required to satisfy the following condition:

For all channels a and b, $\quad \forall s, t[s \overset{a \circ b}{\Rightarrow} t \ \ iff \ \ \exists r(s \overset{a}{\Rightarrow} r \ and \ r \overset{b}{\Rightarrow} t)]$. □

Barwise proposed five test principles (Xerox principle/ Logic as information flow/ Addition of information/ Exhaustive cases/ Contraposition) that should be required as features of information flow (Barwise, 1993). As we regarded channels as perception of agents, to verify 'Logic as information flow' and 'Contraposition' are inadequate because the former can be considered as interpretation in a broader sense by agents, and the latter is the reverse flow of information. In addition, 'Exhaustive cases' can be considered as a combination of serial/ parallel conjunction of multiple channels. Thus, we only pay attention to the following two test principles in this paper.

(1) Xerox Principle If there is a channel $c_1 : \varphi \rightarrow \psi$ and $c_2 : \psi \rightarrow \theta$, then there is a channel $c = c_1 \circ c_2 : \varphi \rightarrow \theta$. □

(2) Addition of Information Assume that $c = c_1 \parallel c_2$ exists. If $s_1 : \varphi \overset{c_1}{\Rightarrow} s_2 : \psi \ and \ s_1 : \varphi\prime \overset{c_2}{\Rightarrow} s_2 : \psi\prime \ then \ s_1 : (\varphi \wedge \varphi\prime) \overset{c}{\Rightarrow} s_2 : (\psi \wedge \psi\prime)$, where $c = c_1 \parallel c_2 \overset{def}{\equiv} \forall s_1, s_2 \ s_1 \overset{c}{\Rightarrow} s_2$ iff $s_1 \overset{c_1}{\Rightarrow} s_2$ and $s_1 \overset{c_2}{\Rightarrow} s_2$. □

4.2 The application of our model to the test principles

We investigate whether these two principles are valid when P is a channel and input is a sentence with only B or no operator at all.

(1) Xerox Principle Suppose the sites $s_1 : \varphi$, $s_2 : \psi$, and $s_3 : \theta$ are $\mathcal{K}_a$, $\mathcal{K}_b$, and $\mathcal{K}_c$ and the channels c_1 and c_2 are both P. We check whether the channel c is P if input is a sentence Φ with no operator(Φ) or $B(B\Phi)$.

Non: valid, because $\mathcal{K}_a \models_S \Phi \overset{P}{\Rightarrow} \mathcal{K}_b \models_S \Phi$ and $\mathcal{K}_b \models_S \Phi \overset{P}{\Rightarrow} \mathcal{K}_c \models_S \Phi$,
then $\mathcal{K}_a \models_S \Phi \overset{P}{\Rightarrow} \mathcal{K}_c \models_S \Phi$.

Belief: valid, because same as **Non**. □

(2) Addition of Information Suppose the sites s_1 and s_2 are $\mathcal{K}_a$ and $\mathcal{K}_b$ and the channels c_1 and c_2 are both P. We check the channel $c = c_1 \circ c_2(c_1 \wedge c_2)$ is P if the input to c_1 and c_2 are sentences Φ and Ψ with no operator(Φ and Ψ) or $B(B\Phi$ and $B\Psi)$.

Non: valid, because
$(\mathcal{K}_a \models_S \Phi \wedge \mathcal{K}_a \models_S \Psi) \supset \mathcal{K}_a \models_S (\Phi \wedge \Psi) \overset{P}{\Rightarrow} \mathcal{K}_b \models_S (\Phi \wedge \Psi) \supset (\mathcal{K}_b \models_S \Phi \wedge \mathcal{K}_b \models_S \Psi)$

Belief: invalid, because $B(\Phi \wedge \Psi) \not\subset (B\Phi \wedge B\Psi)$ then
$(\mathcal{K}_a \models_S B\Phi \wedge \mathcal{K}_a \models_S B\Psi) \supset \mathcal{K}_a \models_S B\Phi \wedge B\Psi$
$\not\supset \mathcal{K}_a \models_S B(\Phi \wedge \Psi) \overset{P}{\Rightarrow} \mathcal{K}_b \models_S B(\Phi \wedge \Psi) \supset \mathcal{K}_b \models_S B\Phi \wedge \mathcal{K}_b \models_S B\Psi$ □

These two test principles are especially important for the following reasons. When we adopt arrow logic in our formalization, as in the following section, the first Xerox principle can be regarded as arrows composition. The second principle concerns multiple channels between agents. Thus, we can consider the problem of *channel bifurcation* and *channel confluence*. In the following subsection, we take a look into the problem of channel bottleneck as a variant of channel bifurcation/confluence, that we often encounter in the real world.

4.3 Channel bottleneck

As we have seen above, the B sentence does not satisfy the addition of information principle. This means that B sentences are vulnerable with regard to conjunction. If the channel does not have enough capacity, B sentences is decomposed to subsentences (see the following formula (1)), although those subsentences can be transmitted via P channels independently (2), they can never be retrieved as the sender's original meaning (4).

$$
\begin{aligned}
B(\Phi \wedge \Psi) \quad &\supset \quad B\Phi \wedge B\Psi && (1)\\
&\overset{P}{\Rightarrow} \quad PB\Phi \wedge PB\Psi && (2)\\
&\supset \quad B\Phi \wedge B\Psi && (3)\\
&\not\supset \quad B(\Phi \wedge \Psi) && (4)
\end{aligned}
$$

Only if the channel have enough capacity to include $P(\Phi \wedge \Psi)$, then the following transmission:

$$B(\Phi \wedge \Psi) \overset{P}{\Rightarrow} PB(\Phi \wedge \Psi) \supset B(\Phi \wedge \Psi)$$

would be available.

4.4 Unreliable channel

Even our simple communication model can suggest several important aspects in ordinary communications on the unreliability of channels. In the previous section, we have generalized any kind of perception as P. However, although S is strongly reliable because an agent would believe what he/she sees, conversation as P is not so.

- The agent b can perceive Φ by seeing

  $$\mathcal{K}_a \models_S \Phi \overset{P}{\Rightarrow} \mathcal{K}_b \models_S \Phi$$

  $$\mathcal{K}_a \models_S B_a\Phi \overset{P}{\Rightarrow} \mathcal{K}_b \models_S B_a\Phi.$$

- The utterance Φ by agent a suggests that $\mathcal{K}_a \models_S \Phi$. However, the veridicality of Φ is only guranteed in $\mathcal{K}_a$, this implies that what the other agent receives might be $\mathcal{B}_a\Phi$ instead of Φ.

  $$\mathcal{K}_a \models_S \Phi \overset{P}{\Rightarrow} \mathcal{K}_b \models_S B_a\Phi.$$

- We have not discussed thus far the reliability of the channel. However, in ordinary conversations, mistransfer of information can happen. Thus, the agent b might receive 'probably' Φ, and it becomes $\mathcal{K}_b \models_S P\Phi$, instead of $\mathcal{K}_b \models_S \Phi$.

  $$\mathcal{K}_a \models_S \Phi \overset{P}{\Rightarrow} \mathcal{K}_b \models_S B_b\Phi.$$

We can define the following three kinds of channel P about these features of modal operators.

Channel 1 (See) $\forall x, y \;\; \mathcal{K}_x \models_S \Phi \stackrel{P_S}{\Rightarrow} \mathcal{K}_y \models_S \Phi$

Channel 2 (Doubtful) $\forall x, y \;\; \mathcal{K}_x \models_S \Phi \stackrel{P_D}{\Rightarrow} \mathcal{K}_y \models_S B_x\Phi$ (The agent y does not believe the agent x.)

Channel 3 (Probable) $\forall x, y \;\; \mathcal{K}_x \models_S \Phi \stackrel{P_P}{\Rightarrow} \mathcal{K}_y \models_S B_y\Phi$ (The agent y does not believe the channel.)

A channel may satisfy both of **Channel 2** and **Channel 3**. In this case,

$$\forall x, y \;\; \mathcal{K}_x \models_S \Phi \stackrel{P_D \circ P_P}{\Rightarrow} \mathcal{K}_y \models_S B_y B_x \Phi.$$

We will return to this problem in terms of arrow logic in the following section.

5. Agent communication as arrows

In this section we formalize the above three types of channels by dynamic arrow logic (van Benthem, 1994; Marx, Polos, and Masuch, 1996) and propositional dynamic logic (van Benthem, 1996). Then we define an unreliable channel B and its iteration. We can use this formalization to analize an agent network or a channel network.

5.1 Logics in short course

First, we introduce dynamic arrow logic and propositional dynamic logic in short, according to (van Benthem, 1994).

Arrow Logic Arrow Frames are tuples (A, C^3, R^2, I^1) with

A	a set of objects ('arrows') carrying three predicates:
C^3x, yz	x is a 'composition' of y and z
R^2x, y	y is a 'reversal' of x
I^1x	x is a 'identity' arrow

Arrow Models M add a propositional valuation V:

$M, x \models p$	iff	$x \in V(p)$
$M, x \models \neg\phi$	iff	*not* $M, x \models \phi$
$M, x \models \phi \wedge \psi$	iff	$M, x \models \phi$ *and* $M, x \models \psi$
$M, x \models \phi \bullet \psi$	iff	*there exist y,z with Cx,yz and* $M, y \models \phi,\ M, z \models \psi$
$M, x \models \phi^\vee$	iff	*there exist y with Rx,y and* $M, y \models \phi$
$M, x \models Id$	iff	Ix

(Note that $\models$ is a model, and is different from support relation $\models_S$).

Dynamic Arrow Logic Dynamic arrow logic adds one infinitary operator to the above language:

$M, x \models \phi^*$ iff *x can be C-decomposed into some finite sentence of arrows satisfying* ϕ *in* M

Propositional Dynamic Logic with Arrows A propositional dynamic logic based on the arrow logic is the two-level system: 'arrow talk' and 'state talk'. Thus, a Boolean propositional language and two mechanisms of interaction are added between the following two components:

a test 'mode' ? taking statements to programs,

a domain 'projection' $\langle\ \rangle$ taking programs to statements.

These operation are complemantary:

$\langle\phi?\rangle \leftrightarrow \phi$

$\langle\pi\rangle? \leftrightarrow \pi$

We will use $\phi, \psi, \cdots$ for state assertions and $\pi, \pi_1, \pi_2, \cdots$ for describing programs in this two-tier system. These interpretations are

$M, x \models \phi?$ iff *there exists some s with Tx,s and* $M, s \models \phi$

$M, s \models \langle\pi\rangle$ iff *there exists some x with Ds,x and* $M, x \models \pi$

where the relation Tx, s means x is an identity arrow for point s and the relation Ds, x means s is the left end-point of the arrow x.

A channel can be considered to be an edge of non-deterministic automaton that is some programs which causes noise on information. The input of this program is information and its output is the information changed by noise.

The following definitions using (van Benthem, 1991) are more elegant.

$M, s \models D\pi$ iff *for some x,* $\Delta s, x$ *and* $M, x \models \pi$

$M, x \models L\phi$ iff *for some s,* $\mathcal{L}s, x$ *and* $M, s \models \phi$

$M, x \models R\phi$ iff *for some s,* $\mathcal{R}s, x$ *and* $M, s \models \phi$

where $\Delta s, x$ says that s is an end-point of the arrow x, $\mathcal{L}s, x$ says that s is a left end-point of the arrow x, and $\mathcal{R}s, x$ says that s is a right end-point of the arrow x intuitively.

These operation are also complemantary:

$DL\phi \leftrightarrow \phi$ and $DR\phi \leftrightarrow \phi$

We can translate the old formulae to the new ones:

$\phi? : L\phi \wedge Id$ and $\langle\pi\rangle : D(\pi \bullet T)$

5.2 Unreliable channels revisited

We can interpret the formula:

$$M, x \models L\phi$$

$$M, x \models R\phi$$

as the information ϕ is valid as long as the arrow x, because ϕ is retrieved on the left hand side of the arrow x as $L\phi$ and on the right hand side as $R\phi$. In a natural way, if we were to formalize channels as arrows, this kind of persistency of ϕ corresponds to the transparent channels just as P_S in section 4.4. However, there is no such channel in the real world. When we send information by a channel, it normaly contains noise.

When we send information by a channel, it contains noise. So we can think if the input information is a sentence ϕ, the output is the same sentence with some model operator $X\phi$. We claim here that this is the very nature of 'unreliable' channel.

5.3 Xerox principle revisited

We show the Xerox Principle for information flow by (Barwise, 1993) in the previous section. As we consider a channel as an arrow, this principle shows that if two arrows (end-point of one arrow is start-point of the other one) are connected, then the compositional function C^3 in arrow logic is defined for these two arrows and compositional arrow exists. Thus, arrow structure has a composition.

5.4 Channel bottleneck revisited

We can redefine the channels P_S, P_D, P_P in terms of arrow logic as follows.

$M, x \models P_S\phi$ iff $M, x \models L\phi$ and $M, x \models R\phi$

$M, x \models P_D\phi$ iff $M, x \models L\phi$ and $M, x \models RB_L\phi$

$M, x \models P_P\phi$ iff $M, x \models L\phi$ and $M, x \models RB_R\phi$

We can integrate the channel 2 (P_D) and the channel 3 (P_P) into the unreliable channel B abstractedly. It is defined as follows.

$M, x \models P_D\phi$ iff $M, x \models L\phi$ and $M, x \models RB\phi$

Intuitively, the unreliable channel is like a program that adds the B operator to the input information non-deterministically.

In this paper the point we wish to stress is that using arrow logic we can distinctly deal with the communication with some modal operators like belief between agents. Therefore we can more clealy analize the behavior of multi-agents system. Then, we define the iteration of this unreliable channel B:

$M, x \models B^*\phi$ iff *x can be C-decomposed into finite sequences $x_0, x_1, \cdots, x_i$ of arrows satisfying $M, x_0 \models L\phi$ and $M, x_0 \models RB\phi$, $M, x_1 \models LB\phi$ and $M, x_1 \models RBB\phi$, $\cdots$,$M, x_i \models LB \cdots B$* and $M, x_i \models RBB \cdots B\phi$

This satisfies the following principles:

Axiom $B\phi \rightarrow B^*\phi$

Axiom $B^*\phi \bullet B^*\phi \rightarrow B^*\phi$

Rule *if $B\phi \rightarrow \alpha$ and $\alpha \bullet \alpha \rightarrow \alpha$ are provable, then so is $B^*\phi \rightarrow \alpha$*

So we can prove the following channel bottle neck proposition about unreliable channel B^*:

$$B^*(\phi \wedge \psi) \underset{\leftarrow}{\not\rightarrow} B^*\phi \wedge B^*\psi$$

5.5 ... and Furthermore

Finally, we consider the further possibility of dynamic arrow logic in analysis of imperfect information flow.

The first issue is the problem of channel order. If we were to communicate each other, splicing multiple pipes consecutively, we would be required to connect them properly. Namely, Xerox principle is valid if two channels are connected in a right order. C^3x, yz does not necessarily imply C^3x, zy, and thus, $B_yB_z\phi$ may be meaningful information while $B_zB_y\phi$ may be a junk. We can consider this kind of order problem from the viewpoint of arrow logic.

The dynamic logic offers a duality between edges and vertices, viz., a site may be expanded into an arrow and an arrow may be compressed into a site. Actually, what we have done and are trying to do is the modelling of agents; in some phases, it is convenient to represent an entangled arrow network as a site, and in other phases, it is required that an internal structure of a site should be decomposed. Thus, we believe that dynamic arrow logic can be a proper tool for agent model.

We have omitted the last principle of Barwise: the problem of reverse information, because we could not imagine $\phi^\vee$ for some ϕ on channel. However, a simple acknowledgment as $\phi^\vee$ can be useful; if an agent can receive an acknowledgment from the recipient for the information he/she issues, he/she can avoid multiple issues of same information. Otherwise, we can reguard $\phi^\vee$ as 'asking-back' for uncertain information. Like these examples, arrow logic has the latent possibility to refine the meanings of those test principles conversely.

6. Discussion

In this paper, first, we investigated the logical features of sentences with B, D, S, and ST's which usually cannot preserve the feature of conjunction, disjunction, and other logical connectives. Thereafter, we formalized the process of communication between two agents, according to Barwise' information flow, as follows: a unit of information Φ exists in a certain agent, say agent a, in a site $\mathcal{K}$ as $\mathcal{K}_a \models_S \Phi$, and this unit of information is carried by the channel P, either by utterance or by seeing. Then, we inspected the applicability of our model by Barwise' test principles, and also we discussed the problem of channel bottleneck that is critically harmful to B sentences. In addition, looking back the generalization from S and ST to P, we distinguished reliable/ doubtful/ probable perception channels. Lastly we defined these channels using arrow logic and propositional dynamic logic, and doutful and probable channels were integrated to unreliable channel and we formalized the channel network as the iteration of the unreliable channel. We showed the problem of channel bottleneck in the channel network.

However, in our approach, there are still problems. Two main issues for the future are as follows:

- The distinguishability between belief and knowledge: these two concepts are rather continuously defined.
- We intentionally mix agent's own belief and noisy information through unreliable channels.

This paper mainly contributes to the formalization of communication with knowledge, belief, and perception. In addition, the relation between belief and uncertain communication channel discussed in the previous section could be applied to the remodeling of multi-agent environment, for example, to control robot behavior, and so on. We think the formalization of multi-agent network or channel network is useful for recovering from the noisy information contents through unreliable channels.

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