View-Based Models

Goal: Explore ways to model the image appearance of objects under a wide range of viewing conditions.

Motivation:

A central question in vision concerns how we represent objects. One simple approach is to let images themselves be the representation.

- We consider the construction of low-dimensional bases for an ensemble of training images of the object(s) in question using principal components analysis (PCA).
- We introduce PCA, its derivation, its properties, and some of its uses.
- We also briefly discuss some variants on the idea, including linear discriminant analysis.

Readings: Sections 22.1–22.3 of Forsyth and Ponce.
Matlab Tutorials: trainEigenEyes.m and detectEigenEyes.m

Template Matching – Straw Man

What if we just stored all images (templates) of the object(s) in each *characteristic view* available (the simplest possible view-based model).

For detection we could compute a *matching score* based on cross-correlation of each template with every image neighbourhood.

Left Eyes Right Eyes

Problems:

- cross-correlation and related detectors are very sensitive to small variations in object pose, lighting, occlusions, and small variations in object shape and appearance.
- we'd certainly need an extremely large training set of images.
- storage and computation costs become unreasonable as the number of objects and views increases.

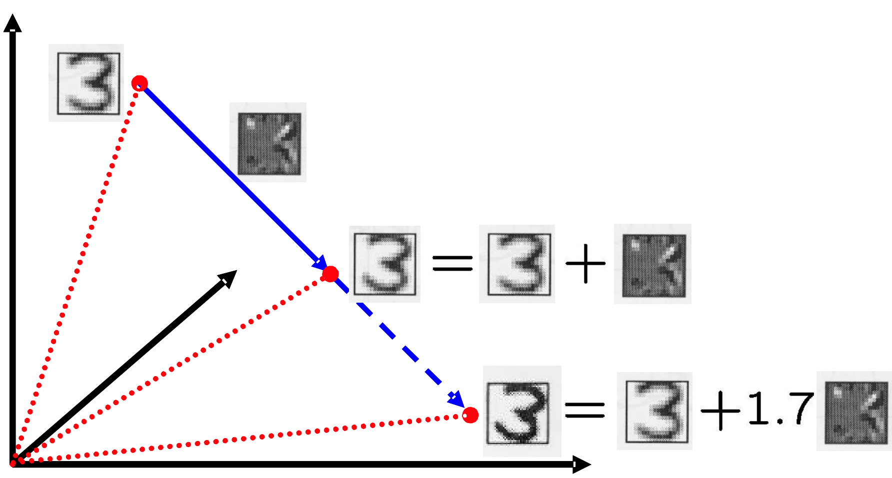
Question: How can we find a more efficient representation for the ensemble of views, and more effectve methods for matching?

Subspace Appearance Models

Idea: Images are not random, especially those of an object, or similar objects, under different viewing conditions.

Rather, than storing every image, we might try to represent the images more effectively, e.g., in a lower dimensional *subspace*.

For example, let's represent each $N \times N$ image as a point in an N^2-dim vector space (e.g., ordering the pixels lexicographically to form the vectors).



(red points denote images, blue vectors denote image differences)

How do we find a low-dimensional basis to accurately model (approximate) each image of the training ensemble (as a linear combination of basis images)?

Linear Subspace Models

We seek a linear basis with which each image in the ensemble is approximated as a linear combination of basis images $b_k(\vec{\mathbf{x}})$

$$I(\vec{\mathbf{x}}) \;\approx\; \sum_{k=1}^{K} a_k\, b_k(\vec{\mathbf{x}}) \tag{1}$$

The *subspace coefficients* $\vec{\mathbf{a}} = (a_1, ..., a_K)$ comprise the representaion.

With some abuse of notation, assuming basis images $b_k(\vec{\mathbf{x}})$ with N^2 pixels, let's define

$\vec{\mathbf{b}}_k$ – an $N^2 \times 1$ vector with pixels arranged in lexicographic order
$\mathbf{B}$ – a matrix with columns $\vec{\mathbf{b}}_k$, *i.e.*, $\mathbf{B} = [\vec{\mathbf{b}}_1, ..., \vec{\mathbf{b}}_K] \in \mathcal{R}^{N^2 \times K}$

With this notation we can rewrite Eq. (1) in matrix algebra as

$$\vec{\mathbf{I}} \;\approx\; \mathbf{B}\,\vec{\mathbf{a}} \tag{2}$$

Choosing The Basis

Orthogonality: Let's assume orthonormal basis functions,

$$\| \vec{\mathbf{b}}_k \|_2 \;=\; 1 \quad , \qquad \vec{\mathbf{b}}_j^{\,T} \vec{\mathbf{b}}_k \;=\; \delta_{jk} \;.$$

Subspace Coefficients: It follows from the linear model in Eq. (2) and the orthogonality of the basis functions that

$$\vec{\mathbf{b}}_k^{\,T} \vec{\mathbf{I}} \;\approx\; \vec{\mathbf{b}}_k^{\,T} \mathbf{B}\,\vec{\mathbf{a}} \;=\; \vec{\mathbf{b}}_k^{\,T} \left[\vec{\mathbf{b}}_1, ..., \vec{\mathbf{b}}_K\right] \vec{\mathbf{a}} \;=\; a_k$$

This selection of coefficients, $\vec{\mathbf{a}} = \mathbf{B}^T \vec{\mathbf{I}}$, minimizes the sum of squared errors (or sum of squared pixel differences, SSD):

$$\min_{\vec{\mathbf{a}} \in \mathcal{R}^K} \| \vec{\mathbf{I}} - \mathbf{B}\,\vec{\mathbf{a}} \|_2^2$$

Basis Images: In order to select the basis functions $\{\vec{\mathbf{b}}_k\}_{k=1}^K$, suppose we have a training set of images

$$\{\vec{\mathbf{I}}_l\}_{l=1}^L \quad , \qquad \text{with } \; L \gg K$$

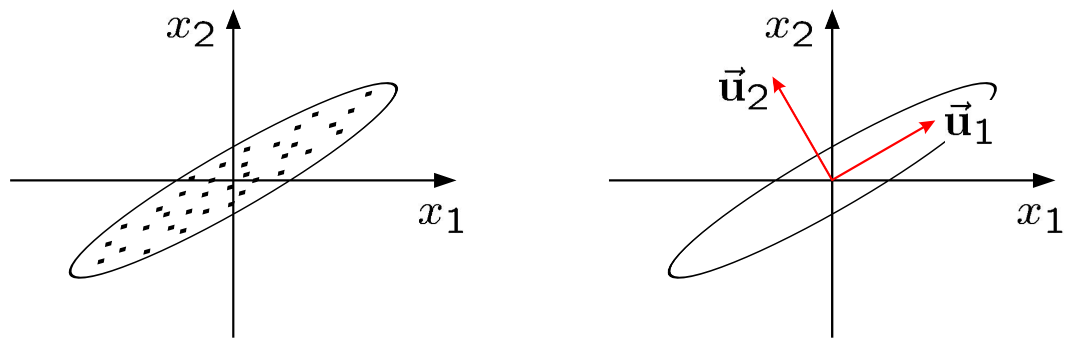
(Let's also assume the images are mean zero. If the mean is nonzero, subtract the mean image, $\frac{1}{L}\sum_l \vec{\mathbf{I}}_l$, from each training image.)

Finally, let's select the basis, $\{\vec{\mathbf{b}}_k\}_{k=1}^K$, to minimize squared reconstruction error:

$$\sum_{l=1}^{L} \min_{\vec{\mathbf{a}}_l} \| \vec{\mathbf{I}}_l - \mathbf{B}\,\vec{\mathbf{a}}_l \|_2^2$$

Intuitions

Example: let's consider a set of images $\{\vec{\mathbf{I}}_l\}_{l=1}^L$, each with only two pixels. So, each image can be viewed as a 2D point, $\vec{\mathbf{I}}_l \in \mathcal{R}^2$.



For a model with only one basis image, what should $\vec{\mathbf{b}}_1$ be?

Approach: Fit an ellipse to the distribution of the image data, and choose $\vec{\mathbf{b}}_1$ to be a unit vector in the direction of the major axis.

Define the ellipse as $\vec{\mathbf{x}}^T C^{-1} \vec{\mathbf{x}} = 1$, where C is the sample covariance matrix of the image data,

$$\mathbf{C} = \frac{1}{L} \sum_{l=1}^{L} \vec{\mathbf{I}}_l \vec{\mathbf{I}}_l^T$$

The eigenvectors of $\mathbf{C}$ provide the major axis, i.e.,

$$\mathbf{C}\,\mathbf{U} = \mathbf{U}\,\mathbf{D}$$

for orthogonal matrix $\mathbf{U} = [\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2]$, and diagonal matrix $\mathbf{D}$ with elements $d_1 \geq d_2 \geq 0$. The direction $\vec{\mathbf{u}}_1$ associated with the largest eigenvalue is the direction of the major axis, so let $\vec{\mathbf{b}}_1 = \vec{\mathbf{u}}_1$.

Principal Components Analysis

Theorem: *(Minimum reconstruction error)* The orthogonal basis $\mathbf{B}$, of rank $K < N^2$, that minimizes the squared reconstruction error over training data, $\{\vec{\mathbf{I}}_l\}_{l=1}^L$, i.e.,

$$\sum_{l=1}^{L} \min_{\vec{\mathbf{a}}_l} \| \vec{\mathbf{I}}_l - \mathbf{B}\,\vec{\mathbf{a}}_l \|_2^2$$

is given by the first K eigenvectors of the data covariance matrix

$$\mathbf{C} = \frac{1}{L} \sum_{l=1}^{L} \vec{\mathbf{I}}_l \, \vec{\mathbf{I}}_l^{\,T} \in \mathcal{R}^{N^2 \times N^2} \quad , \quad \text{for which} \quad \mathbf{C}\,\mathbf{U} = \mathbf{U}\,\mathbf{D}$$

where $\mathbf{U} = [\vec{\mathbf{u}}_1, ..., \vec{\mathbf{u}}_{N^2}]$ is orthogonal, and $\mathbf{D} = \text{diag}(d_1, ..., d_{N^2})$ comprises the eigenvalues of $\mathbf{C}$, with $d_1 \geq d_2 \geq ... \geq d_{N^2}$.

That is, the optimal basis vectors are $\vec{\mathbf{b}}_k = \vec{\mathbf{u}}_k$, for $k = 1...K$. The corresponding basis images $\{b_k(\vec{\mathbf{x}})\}_{k=1}^K$ are often called eigen-images.

Proof: see the derivation below.

Derivation of PCA

To begin, we want to find $\mathbf{B}$ in order to minimize squared error in subspace approximations to the images of the training ensemble.

$$E \;=\; \sum_{l=1}^{L} \min_{\vec{\mathbf{a}}_l} \| \,\vec{\mathbf{I}}_l - \mathbf{B}\,\vec{\mathbf{a}}_l \,\|_2^2$$

Given the assumption that the columns of $\mathbf{B}$ are orthonormal, the optimal coefficients are $\vec{\mathbf{a}}_l = \mathbf{B}^T\vec{\mathbf{I}}_l$, so

$$E \;=\; \sum_{l=1}^{L} \min_{\vec{\mathbf{a}}_l} \| \,\vec{\mathbf{I}}_l - \mathbf{B}\,\vec{\mathbf{a}}_l \,\|_2^2 \;\;=\;\; \| \,\vec{\mathbf{I}}_l - \mathbf{B}\,\mathbf{B}^T\vec{\mathbf{I}}_l \,\|_2^2 \tag{3}$$

Furthermore, writing the each training image as a column in a matrix $\mathbf{A} = \left[\vec{\mathbf{I}}_1, ..., \vec{\mathbf{I}}_L\right]$, we have

$$E \;=\; \sum_{l=1}^{L} \| \,\vec{\mathbf{I}}_l - \mathbf{B}\,\mathbf{B}^T\vec{\mathbf{I}}_l \,\|_2^2 \;\;=\;\; \| \,\mathbf{A} - \mathbf{B}\,\mathbf{B}^T\mathbf{A}\,\|_F^2 \;\;=\; trace\left[\,\mathbf{A}\,\mathbf{A}^T\,\right] - trace\left[\,\mathbf{B}^T\mathbf{A}\,\mathbf{A}^T\mathbf{B}\,\right]$$

You get this last step by expanding the square and noting $\mathbf{B}^T\mathbf{B} = \mathbf{I}_K$, and using the properties of $trace$, e.g., $trace[\mathbf{A}] = trace[\mathbf{A}^T]$, and also $trace[\mathbf{B}^T\mathbf{A}\mathbf{A}^T\mathbf{B}] = trace[\mathbf{A}^T\mathbf{B}\mathbf{B}^T\mathbf{A}]$.
So the minmize the average squared error in the approximation we want to find $\mathbf{B}$ to maximize

$$E' \;=\; trace\left[\,\mathbf{B}^T\mathbf{A}\,\mathbf{A}^T\mathbf{B}\,\right] \tag{4}$$

Now, let's use the fact that for the data covariance, $\mathbf{C}$ we have $\mathbf{C} = \frac{1}{L}\mathbf{A}\,\mathbf{A}^T$. Moreover, as defined above the SVD of $\mathbf{C}$ can be written as $\mathbf{C} = \mathbf{U}\,\mathbf{D}\,\mathbf{U}^T$. So, let's substitute the SVD into E':

$$E' \;=\; trace\left[\,\mathbf{B}^T\mathbf{U}\,\mathbf{D}\,\mathbf{U}^T\mathbf{B}\,\right] \tag{5}$$

where of course $\mathbf{U}$ is orthogonal, and $\mathbf{D}$ is diagonal.

Now we just have to show that we want to choose $\mathbf{B}$ such that the trace strips off the first K elements of $\mathbf{D}$ to maximize E'. Intuitively, note that $\mathbf{B}^T\mathbf{U}$ must be rank K since $\mathbf{B}$ is rank K. And note that the diagonal elements of $\mathbf{D}$ are ordered. Also the trace is invariant under matrix rotation. So, the highest rank K trace we can hope to get is by choosing $\mathbf{B}$ so that, when combined with $\mathbf{U}$ we keep the first K columns of $\mathbf{D}$. That is, the columns of $\mathbf{B}$ should be the first K orthonormal rows of $\mathbf{U}$. We need to make this a little more rigorous, but that's it for now...

Other Properties of PCA

Maximum Variance: The K-D subspace approximation captures the greatest possible variance in the training data.

- For $a_1 = \vec{\mathbf{b}}_1^T \vec{\mathbf{I}}$, the direction $\vec{\mathbf{b}}_1$ that maximizes the variance $\mathrm{E}[a_1^2] = \vec{\mathbf{b}}_1^{\,T} \mathbf{C} \vec{\mathbf{b}}_1$, subject to $\vec{\mathbf{b}}_1^T \vec{\mathbf{b}}_1 = 1$, is the first eigenvector of $\mathbf{C}$.
- The second maximizes $\vec{\mathbf{b}}_2^T \mathbf{C} \vec{\mathbf{b}}_2$ subject to $\vec{\mathbf{b}}_2^T \vec{\mathbf{b}}_2 = 1$ and $\vec{\mathbf{b}}_1^T \vec{\mathbf{b}}_2 = 0$.
- For $a_k = \vec{\mathbf{b}}_k^{\,T} \vec{\mathbf{I}}$, and $\vec{\mathbf{a}} = (a_1, ..., a_K)$, the subspace coefficient covariance is $\mathrm{E}[\vec{\mathbf{a}}\,\vec{\mathbf{a}}^T] = \mathrm{diag}(d_1, ..., d_K)$. That is, the diagonal entries of $\mathbf{D}$ are marginal variances of the subspace coefficients:

$$\sigma_k^2 \;\equiv\; \mathrm{E}[a_k^2] \;=\; d_k \;.$$

  The total variance *captured* in the subspace is $\sum_{k=1}^K \sigma_k^2$.

- Total variance *lost* owing to the subspace projection (i.e., the out-of-subspace variance) is the sum of the last $N^2 - K$ eigenvalues:

$$\frac{1}{L} \sum_{l=1}^{L} \left[\min_{\vec{\mathbf{a}}_l} \| \, \vec{\mathbf{I}}_l - \mathbf{B}\, \vec{\mathbf{a}}_l \, \|_2^2 \right] \;=\; \sum_{k=K+1}^{N^2} \sigma_k^2$$

Decorrelated Coefficients: Since $\mathrm{E}[\vec{\mathbf{a}}\,\vec{\mathbf{a}}^T]$ is diagonal, the subspace coefficients are uncorrelated. If the images are drawn from a Gaussian density, then the coefficients are also statistically independent.

PCA and Singular Value Decomposition

The singular value decomposition of the data matrix $\mathbf{A}$,

$$\mathbf{A} = \left[\vec{\mathbf{I}}_1, ..., \vec{\mathbf{I}}_L\right] \ , \quad \mathbf{A} \in \mathcal{R}^{N^2 \times L} \ , \quad \text{where usually } \ L \ll N^2 \ .$$

is given by

$$\mathbf{A} = \mathbf{U}\,\mathbf{S}\,\mathbf{V}^T$$

where $\mathbf{U} \in \mathcal{R}^{N^2 \times L}$, $\mathbf{S} \in \mathcal{R}^{L \times L}$, $\mathbf{V} \in \mathcal{R}^{L \times L}$. The columns of $\mathbf{U}$ and $\mathbf{V}$ are orthonormal, i.e., $\mathbf{U}^T\mathbf{U} = \mathbf{I}_{L\times L}$ and $\mathbf{V}^T\mathbf{V} = \mathbf{I}_{L\times L}$, and matrix $\mathbf{S}$ is diagonal, $\mathbf{S} = \text{diag}(s_1, ..., s_L)$ where $s_1 \geq s_2 \geq ... \geq s_L \geq 0$.

Theorem: The best rank-K approximation to $\mathbf{A}$ under the Frobenius norm, $\tilde{\mathbf{A}}$, is given by

$$\tilde{\mathbf{A}} = \sum_{k=1}^{K} s_k \vec{\mathbf{u}}_k \vec{\mathbf{u}}_k^{\,T} = \mathbf{B}\,\mathbf{B}^T\mathbf{A} \ , \ \text{ where } \ \min_{rank(\tilde{\mathbf{A}})=K} \|\, \mathbf{A} - \tilde{\mathbf{A}} \,\|_F^2 = \sum_{k=K+1}^{N^2} s_k^2 \ ,$$

and $\mathbf{B} = [\vec{\mathbf{u}}_1, ..., \vec{\mathbf{u}}_K]$. $\tilde{\mathbf{A}}$ is also the best rank-K approximation under the L_2 matrix norm.

What's the relation to PCA and the covariance of the training images?

$$\mathbf{C} = \frac{1}{L}\sum_{l=1}^{L} \vec{\mathbf{I}}_l\, \vec{\mathbf{I}}_l^{\,T} = \frac{1}{L}\mathbf{A}\,\mathbf{A}^T = \frac{1}{L}\mathbf{U}\,\mathbf{S}\,\mathbf{V}^T\mathbf{V}\,\mathbf{S}^T\,\mathbf{U}^T = \frac{1}{L}\mathbf{U}\,\mathbf{S}^2\,\mathbf{U}^T$$

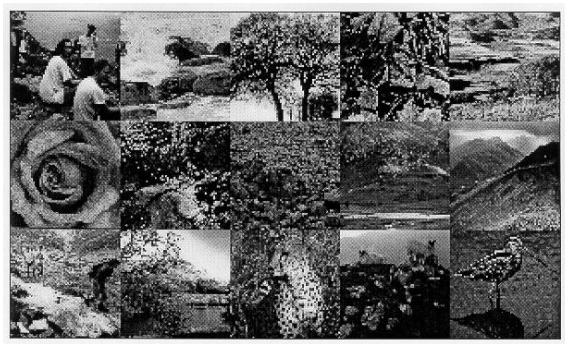
So the squared singular values of $\mathbf{A}$ are proportional to the first L eigenvalues of $\mathbf{C}$:

$$d_k = \begin{cases} \frac{1}{L}\, s_k^2 & \text{for } k = 1, ..., L \\ 0 & \text{for } k > L \end{cases}$$

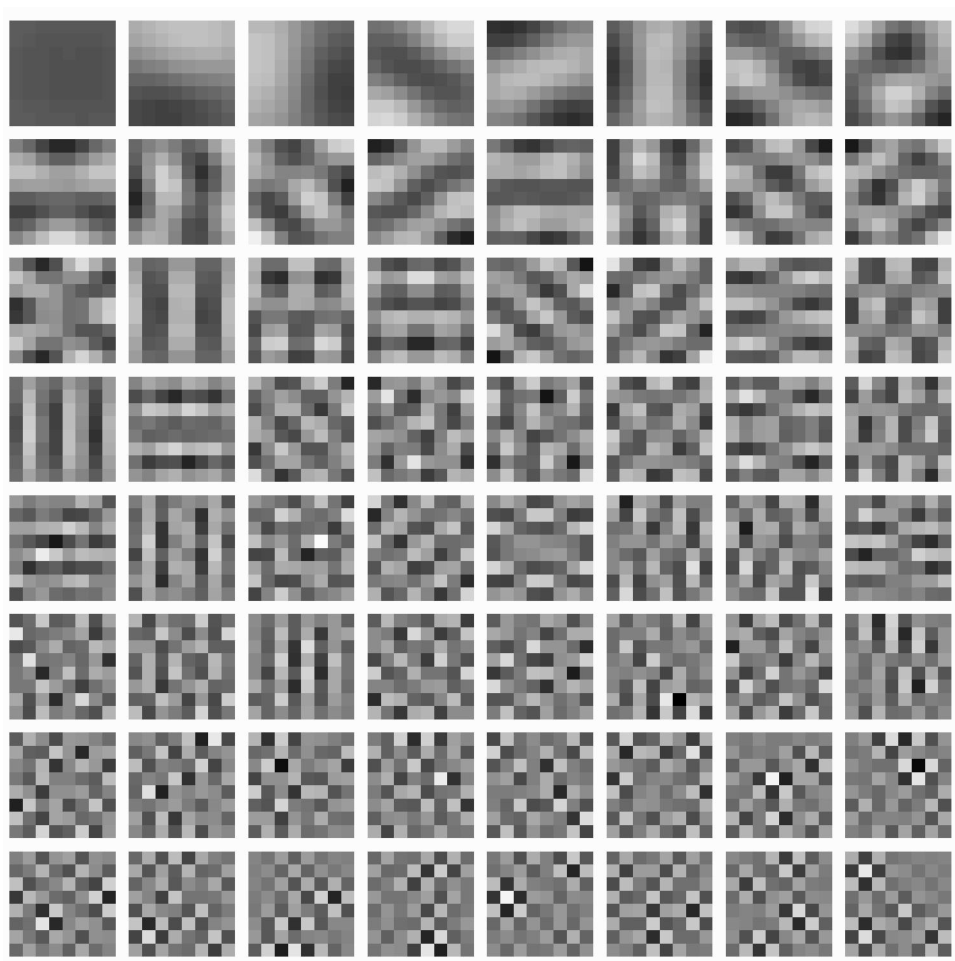
And the singular vectors of $\mathbf{A}$ are just the first L eigenvectors of $\mathbf{C}$.

Eigen-Images for Generic Images?

Fourier components are eigenfunctions of generic image ensembles.



Why? Covariance matrices for stationary processes are Toeplitz.



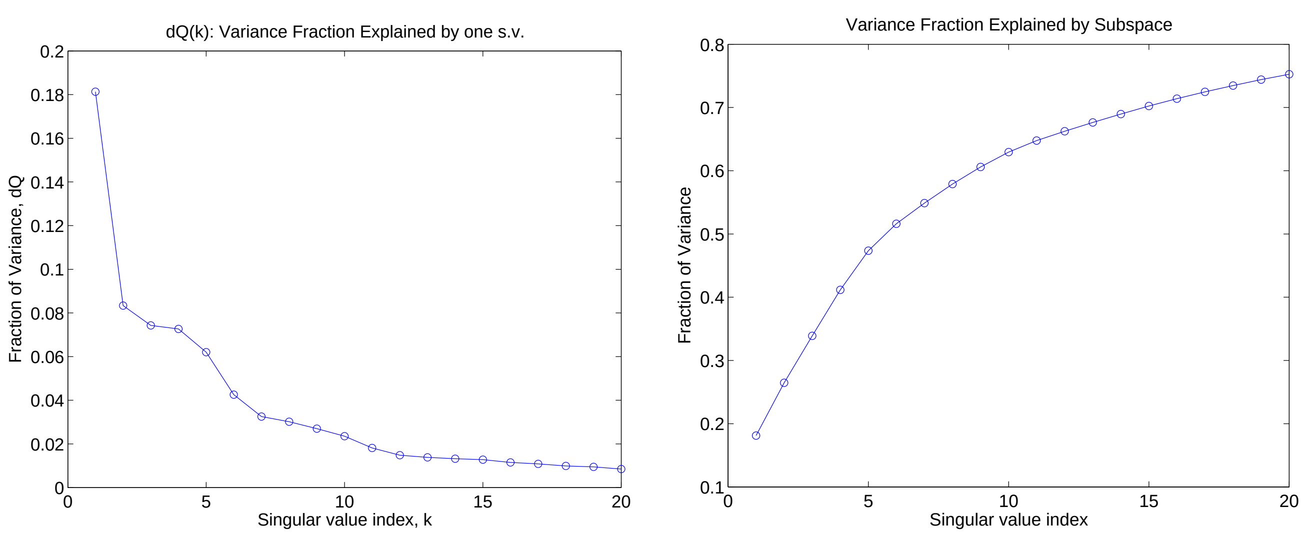
PCA yields unique eigen-images up to rotations of invariant subspaces (e.g., Fourier components with the same marginal variance).

Eye Subspace Model

Subset of 1196 eye images (25×20):

Left Eyes Right Eyes

Variance captured:

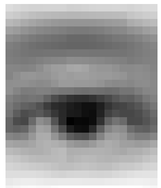


Left plot shows the marginal variance for each principal direction, divided by the total variance in the training data, as a function of the singular value index k.

Right plot shows the fraction of the total variance captured by the subspace $\mathbf{B}$, as a function of the subspace dimension K.

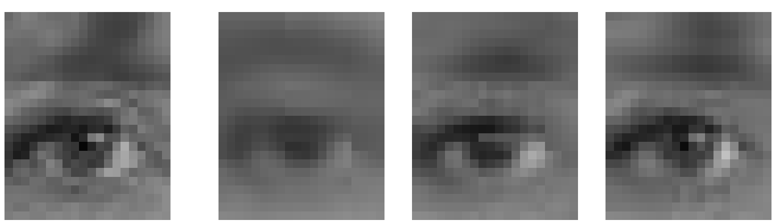
Eye Subspace Model

Mean Eye:

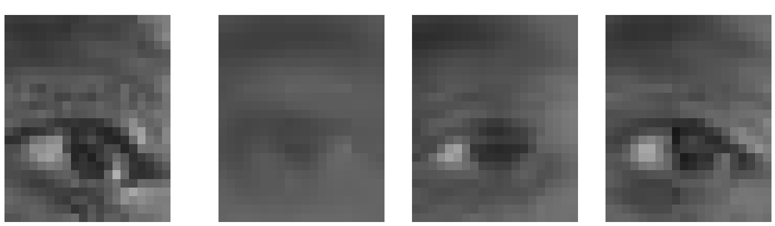


Basis Images $(1-6, \;\text{ and }\; 10\!:\!5\!:\!35)$**:**

Reconstructions (for $K = 5, 20, 50$)**:**



Eye Image | Reconstruction (K = 5) | Reconstruction (K = 20) | Reconstruction (K = 50)



Eye Image | Reconstruction (K = 5) | Reconstruction (K = 20) | Reconstruction (K = 50)

Generative Eye Model

Generative model, $\mathcal{M}$, for random eye images:

$$\vec{\mathbf{I}} = \vec{\mathbf{m}} + \left(\sum_{k=1}^{K} a_k \vec{\mathbf{b}}_k \right) + \vec{\mathbf{e}}$$

where $\vec{\mathbf{m}}$ is the mean eye image, $a_k \sim \mathcal{N}(0, \sigma_k^2)$, σ_k^2 is the sample variance associated with the k^{th} principal direction in the training data, and $\vec{\mathbf{e}} \sim \mathcal{N}(0, \sigma_e^2 \, \mathbf{I}_{N^2})$ where $\sigma_e^2 = \frac{1}{N^2} \sum_{k=K+1}^{N^2} \sigma_k^2$ is the per pixel out-of-subspace variance.

Random Eye Images:

Random draws from generative model (with K = 5, 10, 20, 50, 100, 200)

So the likelihood of an image of an eye given this model $\mathcal{M}$ is

$$p(\vec{\mathbf{I}} \,|\, \mathcal{M}) = \left(\prod_{k=1}^{K} p(a_k | \mathcal{M}) \right) p(\vec{\mathbf{e}} \,|\, \mathcal{M})$$

where

$$p(a_k|\mathcal{M}) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{a_k^2}{2\sigma_k^2}} \quad , \qquad p(\vec{\mathbf{e}} \,|\, \mathcal{M}) = \prod_{j=1}^{N^2} \frac{1}{\sqrt{2\pi}\sigma_e} e^{-\frac{e_j^2}{2\sigma_e^2}} \quad .$$

Eye Detection

The log likelihood of the model is given by

$$
\begin{aligned}
L(\mathcal{M}) \;\equiv\; \log p(\vec{\mathbf{I}}\,|\,\mathcal{M}) \;&=\; \left(\sum_{k=1}^{K} \log p(a_k|\mathcal{M})\right) \;+\; \log p(\vec{\mathbf{e}}\,|\,\mathcal{M}) \\
&=\; \left(\sum_{k=1}^{K} \frac{-a_k^2}{2\sigma_k^2}\right) \;+\; \left(\sum_{j=1}^{N^2} \frac{-e_j^2}{2\sigma_e^2}\right) \;+\; const \\
&\equiv\; S_{in}(\vec{\mathbf{a}}) \;+\; S_{out}(\vec{\mathbf{e}}) \;+\; const
\end{aligned}
$$

Detector:

1. Given an image $\vec{\mathbf{I}}$
2. Compute the subspace coefficients $\vec{\mathbf{a}} = \mathbf{B}^T(\vec{\mathbf{I}} - \vec{\mathbf{m}})$
3. Compute residual $\vec{\mathbf{e}} = \vec{\mathbf{I}} - \vec{\mathbf{m}} - \mathbf{B}\vec{\mathbf{a}}$
4. For $S(\vec{\mathbf{a}}, \vec{\mathbf{e}}) = S_{in}(\vec{\mathbf{a}}) + S_{out}(\vec{\mathbf{e}})$, and a given threshold τ, the image patch is classified as an eye when

$$S(\vec{\mathbf{a}}, \vec{\mathbf{e}}) \;>\; \tau\;.$$

Eye Detection

Terminology:

- true positive = hit
- true negative = correct rejection
- false positive = false alarm (type I error)
- false negative = miss (type II error)

	classified positives	classified negatives	
true examples	true positives, T_{pos}	false negatives, F_{neg}	$N_{pos} = T_{pos} + F_{neg}$
false examples	false positives, F_{pos}	true negatives, T_{neg}	$N_{neg} = F_{pos} + T_{neg}$
	C_{pos}	C_{neg}	N

Definitions:

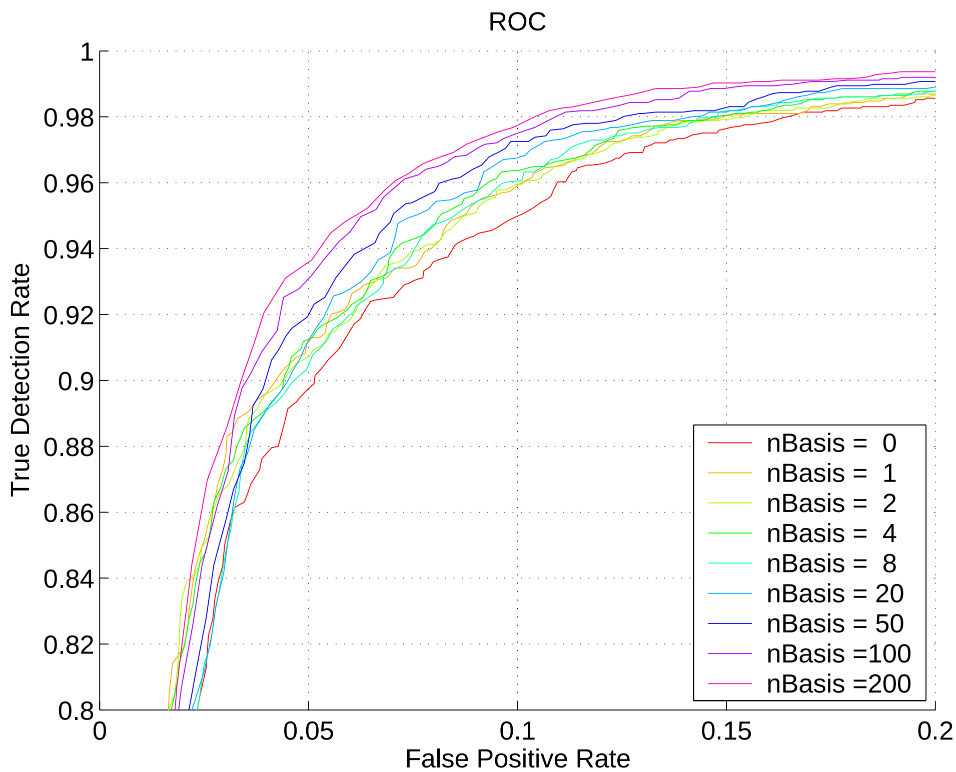
- true positive (hit) rate: $\rho_{tp} = T_{pos}/N_{pos}$ (sensitivity)
  (i.e., what fraction of the true eyes do we find?)
- true negative (reject) rate: $\rho_{tn} = T_{neg}/N_{neg}$ (specificity)
- false positive rate: $\rho_{fp} = F_{pos}/N_{neg} = 1 - \rho_{tn}$ (1 - specificity)

- precision: T_{pos}/C_{pos}
  (i.e., what fraction of positive response are correct hits? ...
  i.e., how noisy is the detector?)
- recall: $\rho_{tp} = T_{pos}/N_{pos}$
  (i.e., what fraction of the true eyes do we actually find?)

Eye Detection

ROC Curves:

- true detection rate (sensitivity) vs false positive rate (1-specificity)
- trade-off (as a function of decision threshold τ) between sensitivity (hit rate) and specificity (responding only to positive cases)



Here the eye images in the test set were different from the those in the training set. Non-eyes were drawn at random from images.

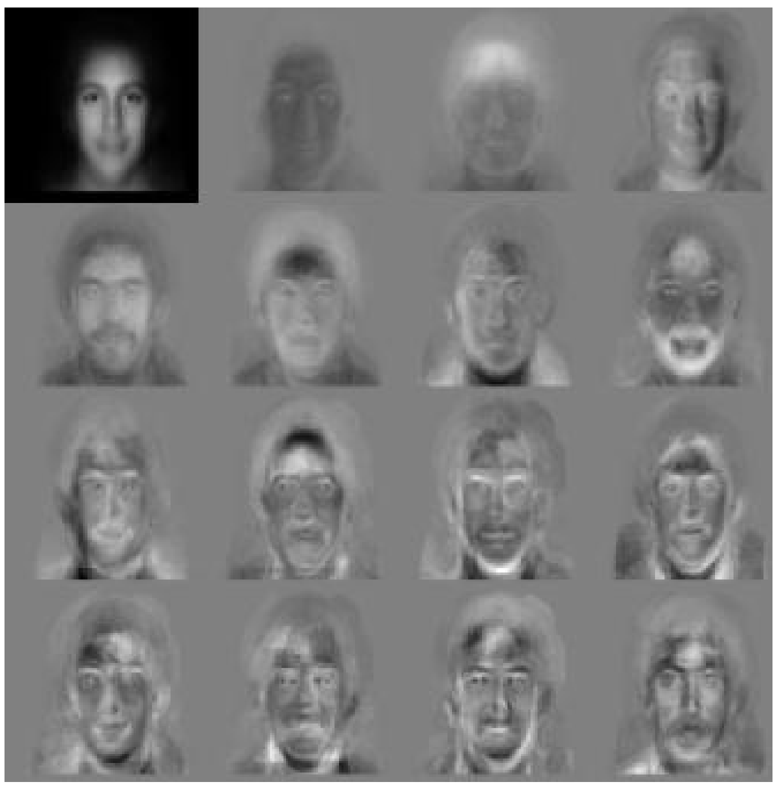
Precision-Recall Curves:

- precision vs true detection rate (sensitivity)
- better whan ROC when the $N_{neg} \gg N_{pos}$, so even a low false positive rate can yield many more false alarms than hits.
- that's why precision divides true hits by total number of hits rather than total number of positives.

Face Detection

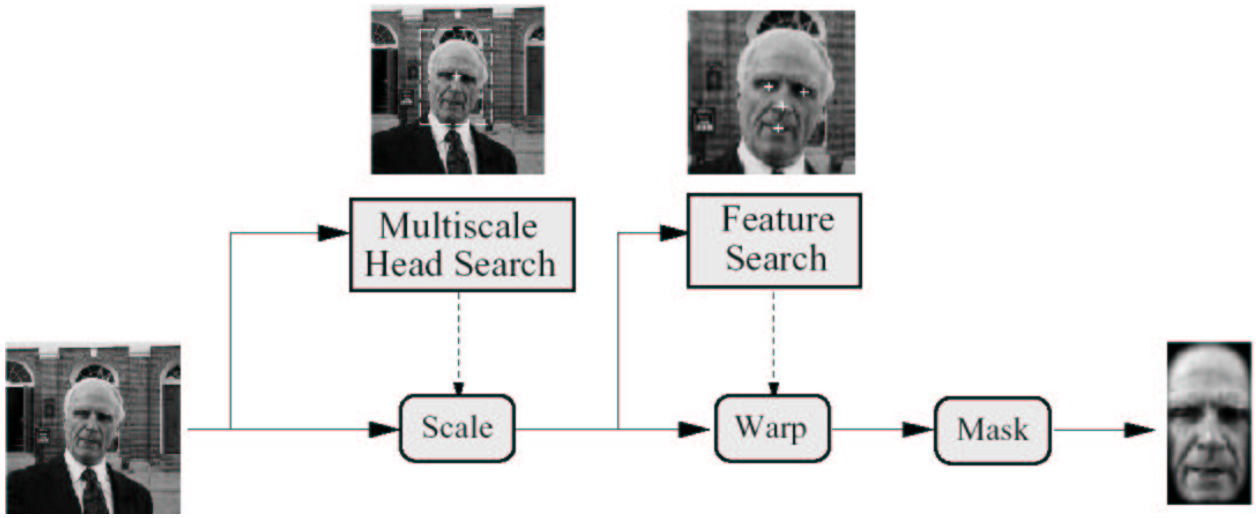
The wide-spread use of PCA for object recognition began with the work Turk and Pentland (1991) for face detection and recognition.

Shown below is the model learned from a collection of frontal faces, normalized for contrast, scale, and orientation, with the backgrounds removed prior to PCA.



Here are the mean image (upper-left) and the first 15 eigen-images. The first three show strong variations caused by illumination. The next few appear to correspond to the occurrence of certain features (hair, hairline, beard, clothing, etc).

Face Detection/Recognition



Moghaddam, Jebara and Pentland (2000): Subspace methods are used for head detection and then feature detection to normalize (warp) the facial region of the image.

Recognition: Are these two images (test and target) the same?

Approach 1: *Single Image Subspace Recognition:*
Project test and target faces onto the face subspace, and look at distance within the subspace.

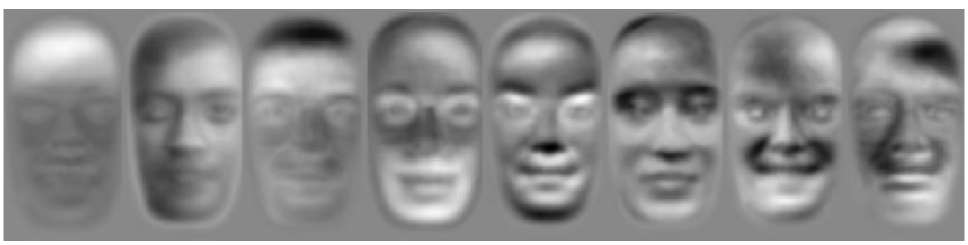
Approach 2: *Intra/Extra-Personal Subspace Recognition:*

- An intra-personal subspace is learned from difference images of the same persion under variation in lighting and expression.
- The extra-personal subspace learned from difference between images of different people under similar conditions.

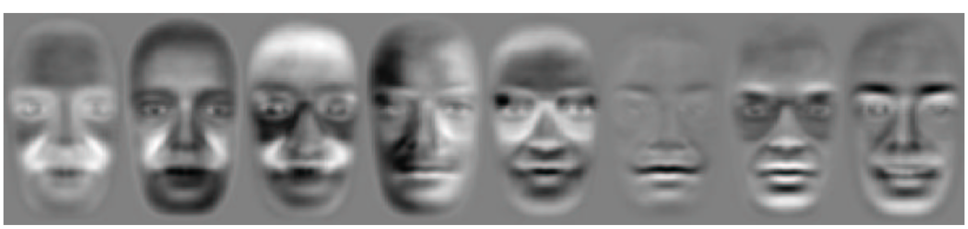
Face Recognition

Example facial pairs for training and testing.

Single Image Face Eigen-Images:



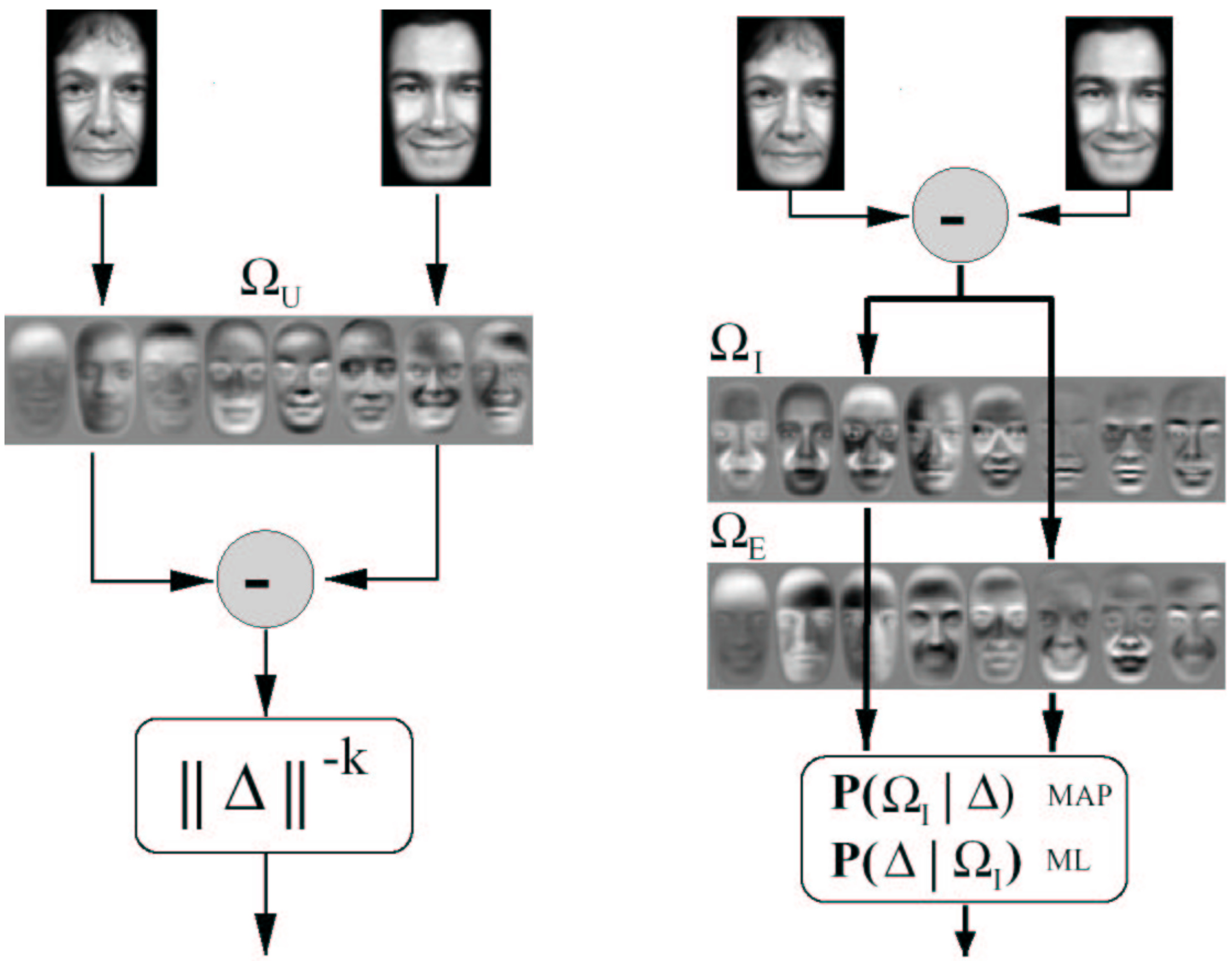
Intra-Personal Face Eigen-Images:



Extra-Personal Face Eigen-Images:

Face Recognition

Single image model Difference image model



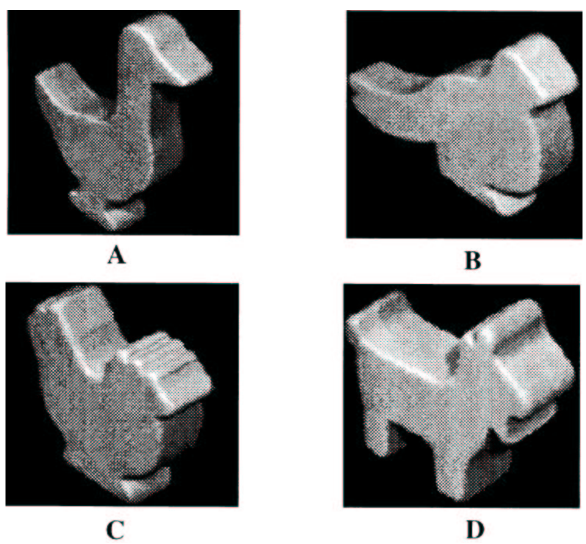
MAP Recognition: What is the probability that the difference between two faces is intra- versus extra-personal?

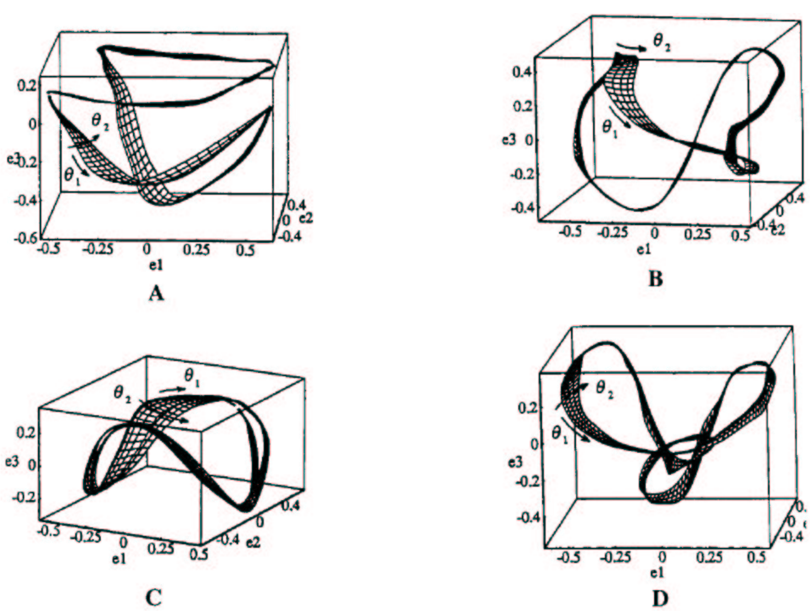
$$p(\Omega_I \,|\, \Delta) = \frac{p(\Delta \,|\, \Omega_I)\, p(\Omega_I)}{p(\Delta \,|\, \Omega_I)\, p(\Omega_I) \;+\; p(\Delta \,|\, \Omega_E)\, p(\Omega_E)}$$

Object Recognition

Murase and Nayar (1995)

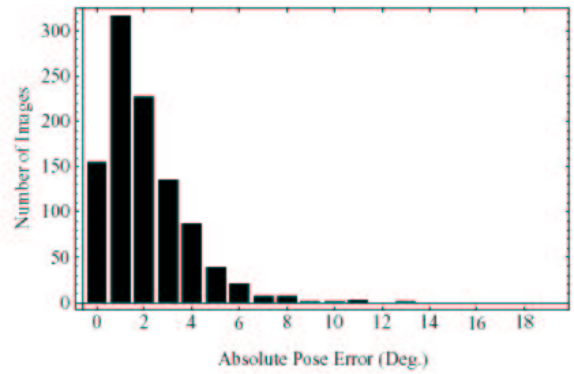
- images of multiple objects, taken from different positions on the viewsphere
- each object occupies a manifold in the subspace (as a function of position on the viewsphere)
- recognition: nearest neighbour assuming dense sampling of object pose variations in the training set.

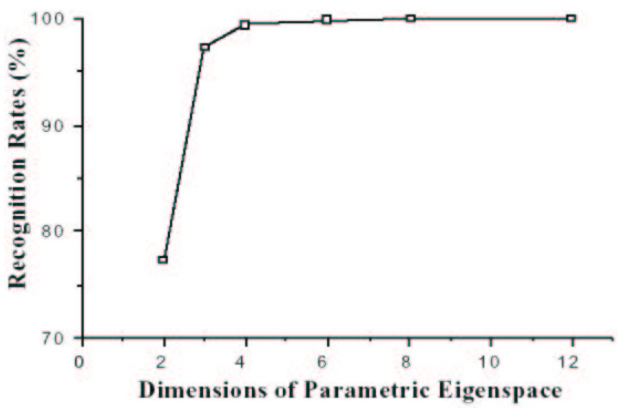


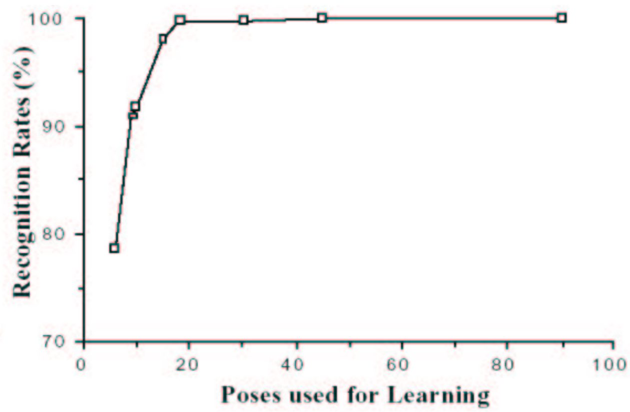


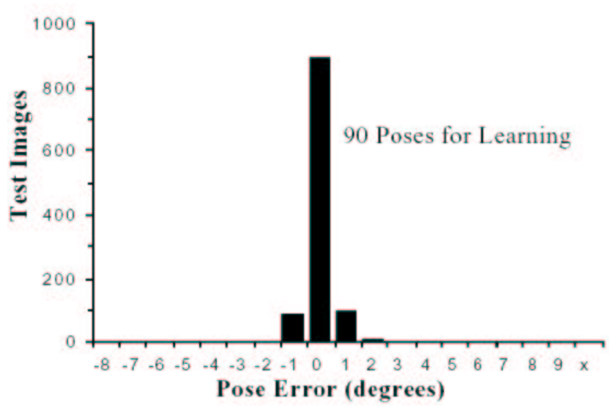
Object Recognition

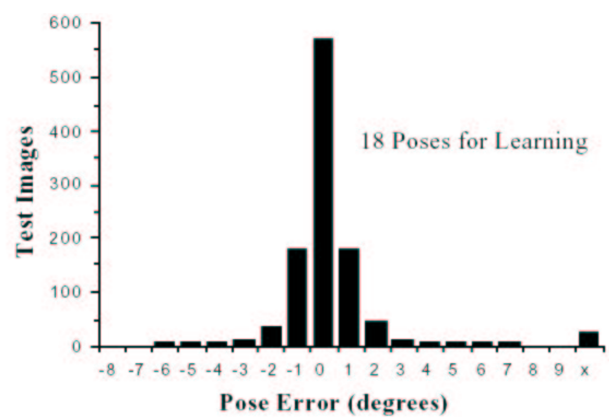
Object Recognition











Linear Discriminant Analysis (LDA)

PCA was originally optimized for good reconstruction, for maximizing variance and decorrelating subspace coefficients, but not for classification.

Linear Discriminant Analysis: (e.g., see [Belhumeur et al, 1997])
Instead of PCA (whose basis directions are chosen to maximize variance), here we choose directions that simultaneously maximize inter-class variation, and minimize intra-class variance.

Given C classes, with means μ_c and N_c training examples for class c, find a basis $\mathbf{B} = [\vec{\mathbf{b}}_1, ..., \vec{\mathbf{b}}_K]$ that maximizes

$$\frac{\mathbf{B}^T C_b \mathbf{B}}{\mathbf{B}^T C_d \mathbf{B}}$$

where

$$\begin{aligned} C_d &= \sum_{c=1}^{C} \sum_{j=1}^{N_c} (\vec{\mathbf{p}}_{c,j} - \mu_c)(\vec{\mathbf{p}}_{c,j} - \mu_c)^T \\ C_b &= \sum_{c} N_c (\hat{\mu} - \mu_c)(\hat{\mu} - \mu_c)^T \end{aligned}$$

and the means are given by $\mu_c = \frac{1}{N_c} \sum_j \vec{\mathbf{p}}_{c,j}$ and $\hat{\mu} = \frac{1}{C} \sum_{c=1}^{C} \mu_c$.

Further Readings

Belhumeur et al (1997) Eigenfaces vs. fisherfaces: Recognition using class specific linear projection. *IEEE Trans. PAMI*, 19(7):711-720

Chennubhotla, C. and Jepson, A.D. (2001) Sparse PCA (S-PCA): Extracting multi-scale structure from data. *Proc. IEEE ICCV*, Vancouver, pp. 641-647.

Golub, G. and van Loan, C. (1984) Matrix Computations. Johns Hopkins Press, Baltimore.

Murase, H. and Nayar, S. (1995) Visual learning and recognition of 3D objects from appearance. *Int. J. Computer Vision* 14:5–24.

M. Black and A. Jepson (1998) EigenTracking: Robust matching and tracking of articulated objects using a view-based representation. *Int. J. Computer Vision* 26(1):63-84.

Moghaddam, B., Jebara, T. and Pentland, A. (2000) Bayesian face recognition. *Pattern Recognition*, 33(11):1771-1782

Turk, M. and Pendland, A. (1991) Face recognition using eigenfaces, *J. Cognitive Neuroscience*, 3(1):71–86.